Problem Set 1 Solutions

1. Ch 1, Problem 1.2

Suppose the U.S. market for corn is competitive, with an upward-sloping supply curve and a downward-sloping demand curve. For each of the following scenarios, illustrate graphically how the exogenous event will contribute to a higher price of corn in the U.S. market.

a. The U.S. Department of Agriculture announces that exports of corn to Taiwan and Japan were “surprisingly bullish,” around 30 percent higher than had been expected.

Surprisingly high export sales means that demand for corn was higher than expected. So the demand curve shifts to the right or up from D1 to D2.

b. Some analysts project that the size of the U.S corn crop will hit a six-year low because of dry weather.

The size of the crop reflects the supply of corn. So if it is at a 6-year low because of dry weather, the weather shock will lead to a upward or left shift in the supply curve.
c. The strengthening of El Nino, the meteorological trend that brings warmer weather to the western coast of South America, reduces corn production outside the United States, thereby increasing foreign countries dependence on the U.S. corn crop.

This is similar to answer a. If foreign countries are demanding more corn from the US, the US demand curve shifts to the right or upwards.

2. Ch 1, Problem 1.4

A firm produces cellular telephone service using equipment and labor. When it uses $E$ machine-hours of equipment and hires $L$ person-ours of labor, it can provide up to $Q$ units of telephone service. The relationship between $Q$, $E$, and $L$ is as follows: $Q = \sqrt{EL}$. The firm must always pay $P_E$ for each machine-hour of equipment it uses and $P_L$ for each person-hour of Labor it hires. Suppose the production manager is told to produce $Q=200$ units of telephone service and that she wants to choose $E$ and $L$ to minimize costs while achieving that production target.

a. What is the objective function for this problem?

The objective function is the relationship the production manager seeks to maximize or minimize. In this example, the production manager wants to minimize costs. The production manager’s costs are given by the following expression:

$$P_E E + P_L L.$$  

Thus, the objective function is $P_E E + P_L L$.

b. What is the constraint?

The constraint will describe the restriction imposed on the production manager. Since the production manager is told to produce $Q = 200$ units of telephone service, the constraint is

$$\sqrt{EL} = 200.$$  

c. Which of the variables ($Q$, $E$, $L$, $P_E$, $P_L$) are exogenous? Which are endogenous? Explain.

The exogenous variables are the ones the production manager takes as given when she makes her decisions. Since she takes the production target ($Q = 200$) as given, $Q$ is exogenous. The prices of equipment ($P_E$) and labor ($P_L$) are also exogenous, since she cannot control these prices. The production manager’s only choices are the
number of machine-hours of equipment \((E)\) to use and the number of person-hours of labor \((L)\) to hire. Therefore, \(E\) and \(L\) are the endogenous variables.

d. Write a statement of the constrained optimization problem.

The statement of the constrained optimization problem is

\[
\min_{(E, L)} P_E E + P_L L
\]

subject to: \(\sqrt{EL} = 200\)

The first line shows that the production manager wants to choose \(E\) and \(L\) to minimize costs. The second line describes the constraint: the production manager must produce 200 units of telephone service.

3. Ch 1, Problem 1.7

The question states that when the price of gasoline outside the U.S increase, the U.S. supply decreases because firms prefer to sell the gasoline elsewhere. The question then asks: How would an increase in the price of gasoline abroad affect the equilibrium price gasoline in the U.S?

Since we are looking at the US market for gasoline, an increase in the price of gasoline abroad is an exogenous shock to the U.S. supply of gasoline. It leads to a shift up or to the left in the supply curve as there is now less supply in the US market. The demand curve for the US market remains the same. This will lead to a price increase in the US market.

4. Ch 1, Problem 1.12

Suppose the supply curve for wool is given by

\[ Q_s = P, \]

where \(Q_s\) is the quantity offered for sale when the price is \(P\). Also suppose the demand curve for wool is given by

\[ Q_d = 10 - P + I, \]

where \(Q_d\) is the quantity of wool demanded when the price is \(P\) and the level of income is \(I\). Assume \(I\) is an exogenous variable.

a. Suppose the level of income is \(I = 20\). Graph the supply and demand relationships, and indicate the equilibrium levels of price and quantity on your graph.
We can solve for the equilibrium levels of price and quantity analytically by setting

\[ Q^s = Q^d \]

\[ P = 10 - P + I \]

quantity supplied equal to quantity demanded:  \( P = 10 - P + 20 \)

\[ 2P = 30 \]

\[ P^* = 15 \]

Substituting \( P^* = 15 \) into the supply curve yields \( Q^* = 15 \).

b. Explain why the market for wool would not be in equilibrium if the price of wool were 18.

If the price of wool were 18, only 12 units of wool would be demanded, but 18 units of wool would be offered for sale. Thus, there would be an excess supply of wool on the market. Some sellers would not find buyers for their wool. To find buyers, these disappointed producers would be willing to sell for less than \( P^* = 15 \). The market price would need to fall to \( P^* = 15 \) to eliminate the excess supply.

c. Explain why the market for wool would not be in equilibrium if the price of wool were 14.

If the price of wool were 14, 16 units of wool would be demanded, but only 14 units of wool would be offered for sale. Thus, there would be an excess demand for wool in the market. Some buyers would be unable to obtain wool. These disappointed buyers would be willing to pay more than 14 for a unit of wool. The market price would need to rise to \( P^* = 15 \) to eliminate the excess demand.
5. Suppose there are only two goods (X and Y) and only two individuals (numbered 1 and 2) in an economy. Let \( P_X \) be the price of good X and \( P_Y \) be the price of good Y. And finally, let \( I_1 \) represent the income of individual 1 and \( I_2 \) the income of individual 2.

Suppose the quantity of good X demanded by individual 1 is given by
\[
X_1 = 10 - 2P_X + 0.01I_1 + 0.4P_Y ,
\]
and the quantity of X demanded by individual 2 is
\[
X_2 = 5 - P_X + 0.02I_2 + 0.2P_Y .
\]

a. What is the market demand function for total X (= \( X_1 + X_2 \)) as a function of \( P_X, I_1, I_2, \) and \( P_Y \)?

The market demand function is, generally, the sum of the two individual demand functions. The exception occurs when the quantity demanded by a particular individual is negative: in this case, you don’t want to add a negative quantity to the total quantity demanded (you want to just add zero). This gives us the following framework for constructing the market demand function:

\[
X = \begin{cases} 
X_1 + X_2 & \text{if } X_1 \geq 0 \text{ and } X_2 \geq 0 \\
X_1 & \text{if } X_1 \geq 0 \text{ and } X_2 < 0 \\
X_2 & \text{if } X_1 < 0 \text{ and } X_2 \geq 0 \\
0 & \text{if } X_1 < 0 \text{ and } X_2 < 0
\end{cases}
\]

After plugging in the formula for \( X_1 \) and \( X_2 \), we obtain

\[
X = \begin{cases} 
15 - 3P_X + 0.6P_Y + 0.01I_1 + 0.02I_2 & \text{if } P_X \leq 5 + 0.2P_Y + 0.005I_1 \\
10 - 2P_X + 0.4P_Y + 0.01I_1 & \text{if } P_X \leq 5 + 0.2P_Y + 0.02I_2 \\
5 - P_X + 0.2P_Y + 0.02I_2 & \text{if } P_X > 5 + 0.2P_Y + 0.005I_1 \\
0 & \text{if } P_X > 5 + 0.2P_Y + 0.02I_2
\end{cases}
\]
b. Graph the two individual demand curves (with $X$ on the horizontal axis and $P_X$ on the vertical axis) for the case $I_1 = 1000$, $I_2 = 1000$, and $P_Y = 10$.

![Demand curves graph]

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c. Using the individual demand curves obtained in part b, graph the market demand curve for total $X$. What is the algebraic equation for this curve?

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The algebraic equation for this curve was derived in part a: after plugging in $I_1 = 1000$, $I_2 = 1000$, and $P_Y = 10$, we obtain

$$X = \begin{cases} 
51 - 3P_X & \text{if } 0 \leq P_X \leq 12 \\
27 - P_X & \text{if } 12 < P_X \leq 27 \\
0 & \text{if } P_X > 27 
\end{cases}$$
6. Suppose the demand for lychees is given by the following equation:

\[ Q^d = 4000 - 100P + 500P_M , \]

where \( P \) is the price of lychees and \( P_M \) is the price of mangoes.

a. What happens to the demand for lychees when the price of mangoes goes up? Are lychees and mangoes substitutes or complements?

The demand for lychees increases when the price of mangoes goes up. Therefore, lychees and mangoes are substitutes.

b. Graph the demand curve for lychees when \( P_M = 2 \).

Now suppose that the quantity of lychees supplied is given by the following equation:

\[ Q' = 1500P - 60R , \]

where \( R \) is the amount of rainfall.

c. On the same graph you drew for part b, graph the supply curve for lychees when \( R = 50 \). Label the equilibrium price and quantity with \( P^* \) and \( Q^* \) respectively.
d. Calculate the equilibrium price and quantity of lychees.

Setting the quantity supplied equal to the quantity demanded, we obtain

\[ Q^d = Q^s \]
\[ 5000 - 100P = 1500P - 3000 \]
\[ 8000 = 1600P \]
\[ P^* = 5 \]

Plugging the equilibrium price \((P^* = 5)\) into the demand curve, we obtain

\[ Q^d = 5000 - 100P \]
\[ Q^* = 4500 \]

e. At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply. Is the demand for lychees elastic, unit elastic, or inelastic? Is the supply of lychees elastic, unit elastic, or inelastic?

The price elasticity of demand is

\[ \varepsilon_{Q,P} = \frac{\partial Q^d}{\partial P} \frac{P^*}{Q^*} \]
\[ = -100 \left( \frac{5}{4500} \right) \]
\[ = -0.1 \]

The demand for lychees is inelastic. The price elasticity of supply is

\[ \varepsilon_{Q,P} = \frac{\partial Q^s}{\partial P} \frac{P^*}{Q^*} \]
\[ = 1500 \left( \frac{5}{4500} \right) \]
\[ = 2.5 \]
The supply of lychees is elastic.

f. At the equilibrium values, calculate the cross-price elasticity of demand for lychees with respect to the price of mangoes. What does the sign of this elasticity tell you about whether lychees and mangoes are substitutes or complements? (Hint: Check to make sure that your answer is consistent with your answer to part a.)

The cross-price elasticity of demand for lychees with respect to the price of mangoes is

$$\varepsilon_{Q,M} = \frac{\partial Q^*}{\partial P_M} \frac{P_M}{Q^*}$$

$$= 500 \left( \frac{2}{4500} \right)$$

$$= 0.2$$

Since the cross-price elasticity of demand is positive, the two goods are substitutes.

7. Consider the demand curve $Q = aP^{-b}$, where $a$ and $b$ are positive constants. Use the formula for price elasticity of demand given in class,

$$\varepsilon_{Q,P} = \frac{\partial Q}{\partial P} \frac{P}{Q},$$

to show that the price elasticity of demand is equal to $-b$ at every point on the demand curve.

We start by calculating the partial derivative of $Q$ with respect to $P$:

$$\frac{\partial Q}{\partial P} = -abP^{-(b+1)}.$$ 

Making the appropriate substitutions using $\frac{\partial Q}{\partial P} = -abP^{-(b+1)}$ and $Q = aP^{-b}$, we obtain

$$\varepsilon_{Q,P} = -abP^{-(b+1)} \left( \frac{P}{aP^{-b}} \right)$$

$$= -abP^{-(b+1)} \left( \frac{1}{aP^{-(b+1)}} \right)$$

$$= -b$$

8. Ch 2, Problem 2.18

In Metropolis only taxi cab and privately owned automobiles are allowed to use the highway between the airport and downtown. The market for taxi cab service
is competitive. There is a special lane for taxicabs, so taxis are always able to travel at 55 miles per hour. The demand for trips by taxi cabs depends on the taxi fare $P$, the average speed of a trip by private automobile on the highway $E$, and the price of gasoline $G$. The number of trips supplied by taxi cabs will depend on the taxi fare and the price of gasoline.

b. Suppose the demand for trips by taxi is given by the equation
\[ Q^d = 1000 + 50G - 4E - 400P. \]
The supply of trips by taxi is given by the equation
\[ Q^s = 200 - 30G + 100P. \]
On a graph draw the supply and demand curves for trips by taxi when $G=4$ and $E=30$. Find equilibrium taxi fare.

To find the equilibrium taxi fare we set $Q^d = Q^s$
\[
200 - 30G + 100P = 1000 + 50G - 4E - 400P
\]
\[
500P = 800 + 80G - 4E
\]
\[
P = \frac{800 + 80G - 4E}{500} = \frac{800 + 80(4) - 4(30)}{500} = \frac{800 + 320 - 120}{500} = 2
\]

c. Solve for equilibrium taxi fare in a general case, that is, when you do not know $G$ and $E$. Show how the equilibrium fare changes as $G$ and $E$ changes.

For part b, we figured out the general case in the 3rd line.
\[
P = \frac{800 + 80G - 4E}{500}
\]
We can see from this equation that that as $G$, the price of gasoline goes up, the equilibrium price of a taxi fare will go up. And as $E$, average speed of private cars goes up, the price of the trip will go down. We can get more precise measures on how exactly the price changes with respect to $G$ and $E$ by using calculus as shown below.

\[
\frac{\partial P}{\partial G} = \frac{80}{500} = \frac{8}{50} = \frac{4}{25} = 0.16
\]
\[
\frac{\partial P}{\partial E} = -\frac{4}{500} = -.008
\]