Problem Set - Ch 1 /2
Chapter 1 Solutions

1. **Ch 1, Problem 1.4**

   A firm produces cellular telephone service using equipment and labor. When it uses E machine-hours of equipment and hires L person-ours of labor, it can provide up to Q unites of telephone service. The relationship between Q, E, and L is as follows: $Q = \sqrt{EL}$. The firm must always pay P_E for each machine-hour of equipment it uses and P_L for each person-hour of Labor it hires. Suppose the production manager is told to produce *Q=200* units of telephone service and that she wants to choose E and L to minimize costs while achieving that production target.

   a. **What is the objective function for this problem?**

      The objective function is the relationship the production manager seeks to maximize or minimize. In this example, the production manager wants to minimize costs. The production manager's costs are given by the following expression:

      $$P_E E + P_L L .$$

      Thus, the objective function is $P_E E + P_L L$.

   b. **What is the constraint?**

      The constraint will describe the restriction imposed on the production manager. Since the production manager is told to produce $Q = 200$ units of telephone service, the constraint is

      $$\sqrt{EL} = 200.$$

   c. **Which of the variables (Q, E, L, P_E, P_L) are exogenous? Which are endogenous? Explain.**

      The exogenous variables are the ones the production manager takes as given when she makes her decisions. Since she takes the production target ($Q = 200$) as given, Q is exogenous. The prices of equipment (P_E) and labor (P_L) are also exogenous, since she cannot control these prices. The production manager's only choices are the number of machine-hours of equipment (E) to use and the number of person-hours of labor (L) to hire. Therefore, E and L are the endogenous variables.

d. Write a statement of the constrained optimization problem.

The statement of the constrained optimization problem is

$$\min_{(E,L)} P_E E + P_L L$$

$$\text{subject to: } \sqrt{EL} = 200$$

The first line shows that the production manager wants to choose E and L to minimize costs. The second line describes the constraint: the production manager must produce 200 units of telephone service.

2. Ch 1, Problem 1.12

Suppose the supply curve for wool is given by

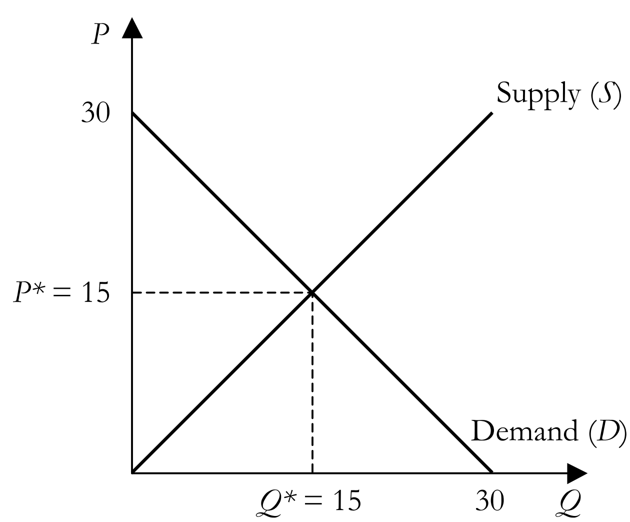
$$Q^s = P,$$

where Q^s is the quantity offered for sale when the price is P. Also suppose the demand curve for wool is given by

$$Q^d = 10 - P + I,$$

where Q^d is the quantity of wool demanded when the price is P and the level of income is I. Assume I is an exogenous variable.

a. Suppose the level of income is $I = 20$. Graph the supply and demand relationships, and indicate the equilibrium levels of price and quantity on your graph.



We can solve for the equilibrium levels of price and quantity analytically by setting quantity supplied equal to quantity demanded:

$$Q^s = Q^d$$

$$P = 10 - P + I$$

$$P = 10 - P + 20$$

$$2P = 30$$

$$P^* = 15$$

Substituting $P^* = 15$ into the supply curve yields $Q^* = 15$.

b. Explain why the market for wool would not be in equilibrium if the price of wool were 18.

If the price of wool were 18, only 12 units of wool would be demanded, but 18 units of wool would be offered for sale. Thus, there would be an excess supply of wool on the market. Some sellers would not find buyers for their wool. To find buyers, these disappointed producers would be willing to sell for less than $P^* = 15$. The market price would need to fall to $P^* = 15$ to eliminate the excess supply.

c. Explain why the market for wool would not be in equilibrium if the price of wool were 14.

If the price of wool were 14, 16 units of wool would be demanded, but only 14 units of wool would be offered for sale. Thus, there would be an excess demand for wool in the market. Some buyers would be unable to obtain wool. These disappointed buyers would be willing to pay more than 14 for a unit of wool. The market price would need to rise to $P^* = 15$ to eliminate the excess demand.

Chapter 2 Solutions

3. **Ch 2, Problem 2.1**

   The demand for beer in Japan is given by the following equation: $Q^d = 700 - 2P - P_N + 0.1I$, where P is the price of beer, P_N is the price of nuts, and I is average consumer income. Assume B is a normal good.

   a) What happens to the demand for beer when the price of nuts goes up? Are beer and nuts demand substitutes or demand complements?

   The sign in front of the prince of nuts, Pn, is negative. This means when the price of nuts goes up, the beer quantity demanded falls for all levels of price (demand shifts left). Beer and nuts are demand complements.

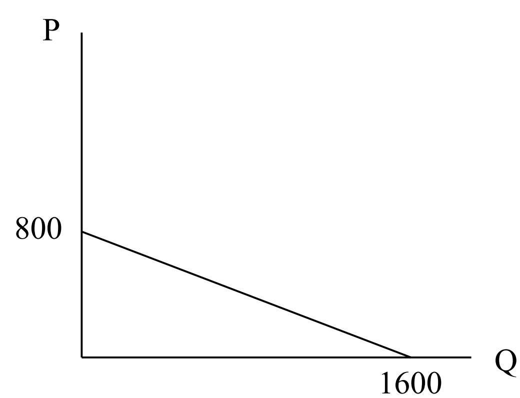
   b) What happens to the demand for beer when average consumer income rises?

   The sign in front of income, I, is positive. This means when income rises, quantity demanded increases for all levels of price (demand shifts rightward).

   c) Graph the demand curve for beer when P_N = 100 and I = 10, 000.

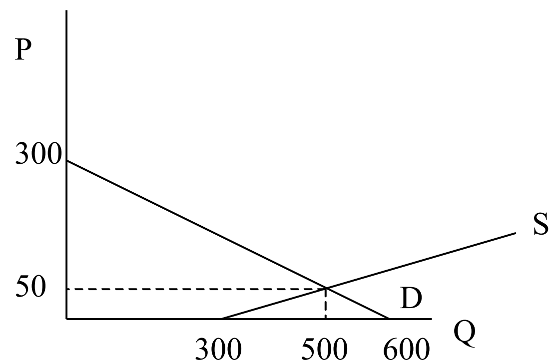
   Now: $Q^d = 700 - 2P - 100 + 0.1 * 10{,}000 = 1{,}600 - 2P \Rightarrow P = 800 - 0.5\,Q^d$

   So when Q^d or Q is zero P=800, When P=0, Q^d or Q is 1600.



4. Ch 2, Problem 2.3
The demand and supply curves for coffee are given by $Q^d = 600 - 2P$ and $Q^s = 300 + 4P$.

a) Plot the supply and demand curves on a graph and show where the equilibrium occurs.



b) Using algebra, determine the market equilibrium price and quantity of coffee. Indicate the equilibrium price and quantity on the graph in part a.

$$600 - 2P = 300 + 4P$$
$$300 = 6P$$
$$50 = P$$

Plugging $P = 50$ back into either the supply or demand equation yields $Q = 500$.

5. Ch 2, Problem 2.13
Consider a linear demand curve, $Q = 350 - 7P$.

a) Derive the inverse demand curve corresponding to this demand curve.

$$Q = 350 - 7P$$
$$7P = 350 - Q$$
$$P = 50 - \tfrac{1}{7}Q$$

b) What is the choke price?
The choke price occurs at the point where $Q = 0$. Setting $Q = 0$ in the inverse demand equation above yields $P = 50$.

c) What is the price elasticity of demand at $P = 50$?

a. At $P = 50$, the choke price, the elasticity will approach negative infinity.

6. Ch 2, Problem 2.17

Consider the following demand and supply relationships in the market for golf balls:

$$Q^d = 90 - 2P - 2T \text{ and}$$
$$Q^s = -9 + 5P - 2.5R,$$

where T is the price of titanium, a metal used to make golf clubs, and R is the price of rubber.

a) If $R = 2$ and $T = 10$, calculate the equilibrium price and quantity of golf balls.

Substituting the values of R and T, we get

$$Demand: Q^d = 70 - 2P$$
$$Supply: Q^s = -14 + 5P$$

In equilibrium, $70 - 2P = -14 + 5P$, which implies that $P = 12$. Substituting this value back, $Q = 46$.

b) At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply.

$$\text{Elasticity of Demand} = \frac{\partial Q^d}{\partial P} * \frac{P}{Q} = -2 * \frac{12}{46} = -0.52$$
$$\text{Elasticity of Supply} = \frac{\partial Q^s}{\partial P} * \frac{P}{Q} = 5 * \frac{12}{46} = 1.30$$

c) At the equilibrium values, calculate the cross-price elasticity of demand for golf balls with respect to the price of titanium. What does the sign of this elasticity tell you about whether golf balls and titanium are substitutes or complements?

$\varepsilon_{golf,titanium} = -2(\frac{10}{46}) = -0.43$. The negative sign indicates that titanium and golf balls are complements, i.e., when the price of titanium goes up the demand for golf balls decreases.

d)

7. Suppose the demand for lychees is given by the following equation:

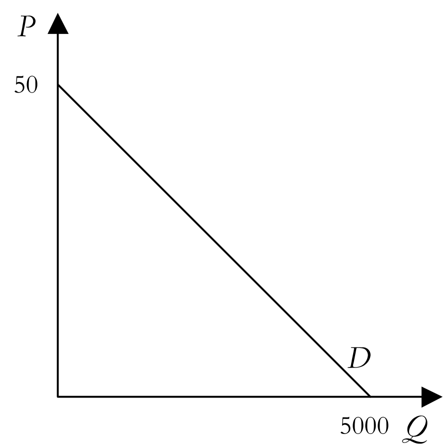
$$Q^d = 4000 - 100P + 500P_M,$$

where P is the price of lychees and P_M is the price of mangoes.

a. What happens to the demand for lychees when the price of mangoes goes up? Are lychees and mangoes substitutes or complements?

The demand for lychees increases when the price of mangoes goes up. Therefore, lychees and mangoes are substitutes.

b. Graph the demand curve for lychees when $P_M = 2$.

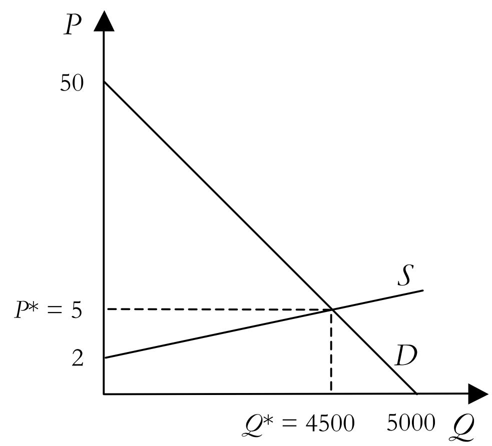


Now suppose that the quantity of lychees supplied is given by the following equation:

$$Q^s = 1500P - 60R,$$

where R is the amount of rainfall.

c. On the same graph you drew for part b, graph the supply curve for lychees when $R = 50$. Label the equilibrium price and quantity with P^* and Q^* respectively.



d. Calculate the equilibrium price and quantity of lychees.

Setting the quantity supplied equal to the quantity demanded, we obtain

$$
\begin{aligned}
Q^d &= Q^s \\
5000 - 100P &= 1500P - 3000 \\
8000 &= 1600P \\
P^* &= 5
\end{aligned}
$$

Plugging the equilibrium price ($P^* = 5$) into the demand curve, we obtain

$$
\begin{aligned}
Q^d &= 5000 - 100P \\
Q^* &= 4500
\end{aligned}
$$

e. At the equilibrium values, calculate the price elasticity of demand and the price elasticity of supply. Is the demand for lychees elastic, unit elastic, or inelastic? Is the supply of lychees elastic, unit elastic, or inelastic?

The price elasticity of demand is

$$
\begin{aligned}
\in_{Q,P} &= \frac{\partial Q^d}{\partial P}\frac{P^*}{Q^*} \\
&= -100\left(\frac{5}{4500}\right) \\
&= -0.\overline{1}
\end{aligned}
$$

The demand for lychees is inelastic. The price elasticity of supply is

$$
\begin{aligned}
\in_{Q^s,P} &= \frac{\partial Q^s}{\partial P}\frac{P^*}{Q^*} \\
&= 1500\left(\frac{5}{4500}\right) \\
&= 1.\overline{6}
\end{aligned}
$$

The supply of lychees is elastic.

f. At the equilibrium values, calculate the cross-price elasticity of demand for lychees with respect to the price of mangoes. What does the sign of this elasticity tell you about whether lychees and mangoes are substitutes or complements? (*Hint:* Check to make sure that your answer is consistent with your answer to part a.)

The cross-price elasticity of demand for lychees with respect to the price of mangoes is

$$\begin{aligned}
\in_{Q,P_M} &= \frac{\partial Q^d}{\partial P_M}\frac{P_M}{Q^*} \\
&= 500\left(\frac{2}{4500}\right) \\
&= 0.\overline{2}
\end{aligned}$$

Since the cross-price elasticity of demand is positive, the two goods are substitutes.