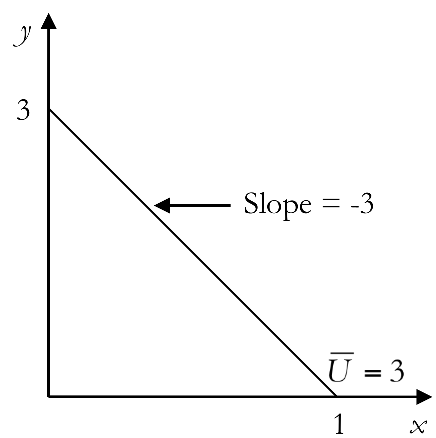
Problem Set – Chapter 3 Solutions

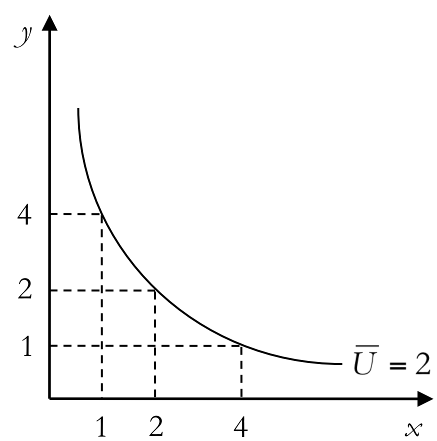
1. Graph a typical indifference curve for the following utility functions and determine whether they obey the assumption of diminishing *MRS*:

a. $U(x,y) = 3x + y$



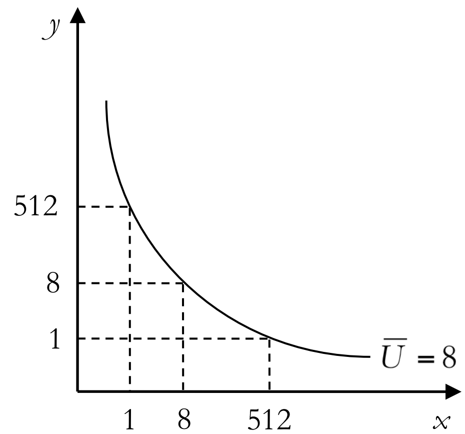
Since the indifference curves are not bowed towards the origin, they do not obey the assumption of diminishing *MRS*.

b. $U(x,y) = \sqrt{x\,y}$



Since the indifference curves are bowed towards the origin, they do obey the assumption of diminishing *MRS*. Alternatively, we know MUx and MUy are both positive. So when quantity of X increases, quantity of Y must decrease. The MRSxy = Y/X . So as X increase, the denominator gets bigger and MRS decreases. As X increase, Y decreases and the numerator gets smaller so MRS decreases. Both these effects work so that as X increase MRS decreasing.

c. $U(x, y) = x^{2/3} y^{1/3}$



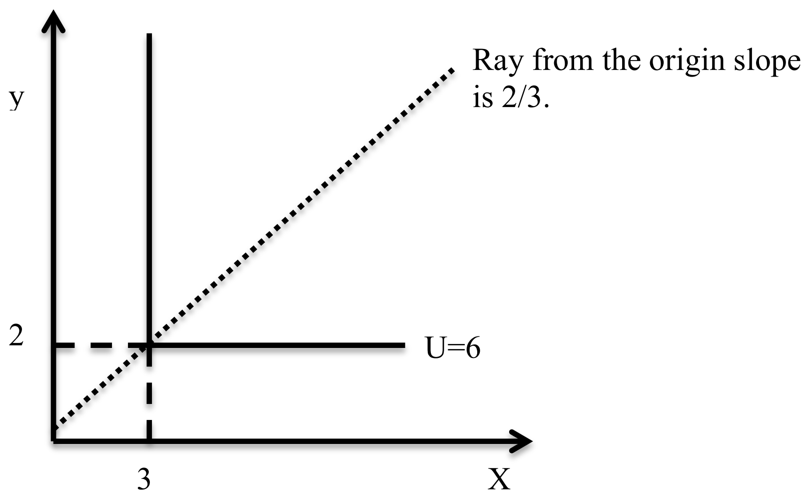
Since the indifference curves are bowed towards the origin, they do obey the assumption of diminishing *MRS*.

d. $U(x, y)$ =min(2X, 3Y)

This is an example of perfect complements. The MRS is undefined at the vertex where 2X=3Y. But lets graph the indifference curve, remember they L shaped. We need to find the corner point. To do this set the two elements of in the utility function equal to each other so there is no extra X or Y being consumed that gives no extra utility.

2X=3Y
rearrange

Y=2X/3 – so ray from original which goes through all the corners of the L has to have the slope 2/3. The indifference curve is for when utility is 6.



2. **Suppose a consumer's preferences for two goods can be represented by the Cobb-Douglas utility function $U(x, y) = A\, x^{\alpha} y^{\beta}$, where A, α, and β are positive constants.**

   a. **What is $MRS_{x,y}$?**

   We begin by calculating the marginal utilities with respect to x and y:

$$MU_x = \frac{\partial U(x, y)}{\partial x} = \alpha A x^{\alpha-1} y^{\beta} \qquad MU_y = \frac{\partial U(x, y)}{\partial y} = \beta A x^{\alpha} y^{\beta-1}$$

   We can then use these marginal utilities to obtain $MRS_{x,y}$:

$$\begin{aligned} MRS_{x,y} &= \frac{MU_x}{MU_y} \\ &= \frac{\alpha A x^{\alpha-1} y^{\beta}}{\beta A x^{\alpha} y^{\beta-1}}. \\ &= \frac{\alpha}{\beta} \frac{y}{x} \end{aligned}$$

b. Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

To determine this, we need to substitute for y using the equation of the indifference curve so as to have $MRS_{x,y}$ expressed solely in terms of x.

The equation of the indifference curve is

$$\overline{U} = Ax^{\alpha}y^{\beta},$$

where $\overline{U}$ represents a constant level of utility. Solving this equation for y gives us

$$y^{\beta} = \frac{\overline{U}}{Ax^{\alpha}}$$

$$y = \left(\frac{\overline{U}}{Ax^{\alpha}}\right)^{\frac{1}{\beta}}$$

$$y = \frac{\overline{U}^{\frac{1}{\beta}}}{A^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}}}$$

Substituting for y in our expression for $MRS_{x,y}$ yields

$$MRS_{x,y} = \frac{\alpha}{\beta}\frac{y}{x}$$

$$= \frac{\alpha}{\beta}\frac{1}{x}\left(\frac{\overline{U}^{\frac{1}{\beta}}}{A^{\frac{1}{\beta}}x^{\frac{\alpha}{\beta}}}\right)$$

$$= \frac{\alpha}{\beta}\left(\frac{\overline{U}}{A}\right)^{\frac{1}{\beta}}\left(\frac{1}{x^{1+\frac{\alpha}{\beta}}}\right)$$

Since A, α, and β are positive constants, the first two terms in the equation above are also positive and constant. Moreover, the exponent on x, $1+\alpha/\beta$, is also positive and constant. Therefore, as x increases, $MRS_{x,y}$ decreases.

That is, $MRS_{x,y}$ is diminishing.

c. On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve. Indicate on your graph whether the indifference curve will intersect either or both axes.

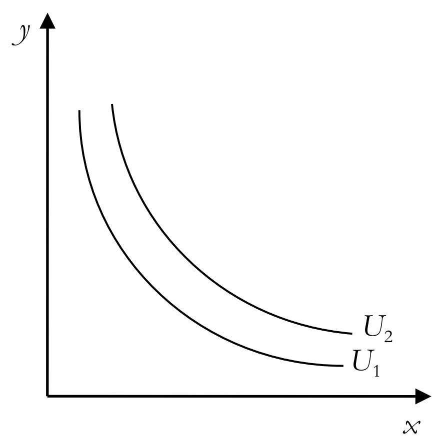
We know "more is better" because MU_x and MU_y are both positive; therefore, the indifference curves must be downward sloping. Moreover, we determined in part b that $MRS_{x,y}$ is diminishing; therefore, the indifference curves must be bowed in towards the origin.

And finally, recall that the equation of a typical indifference curve is given by

$$\overline{U} = Ax^{\alpha} y^{\beta},$$

where $\overline{U}$ represents a constant level of utility. Since for any $\overline{U} > 0$, it cannot be the case that either x or y equals zero, the indifference curves do not intersect either axis.

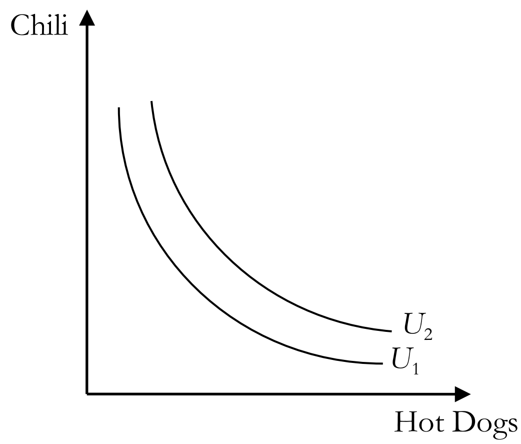
These three observations indicate that the indifference map must be as follows:



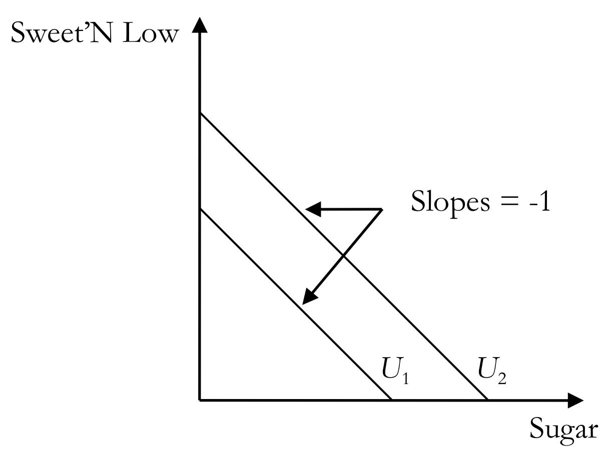
3. **Ch 3, Problem 3.6**

   For the following sets of goods draw two indifference curves, U_1 and U_2, with $U_2 > U_1$. Draw each graph placing the amount of the first good on the horizontal axis.

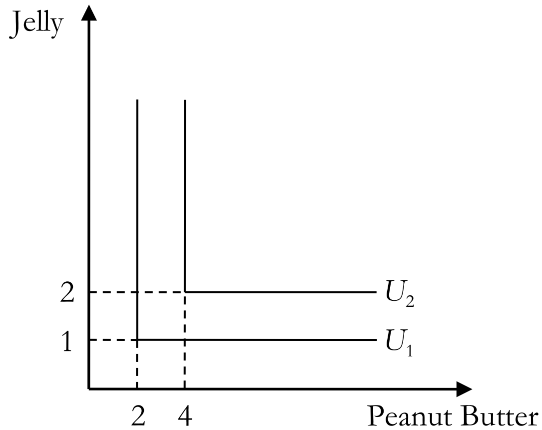
   a. **Hot dogs and chili (the consumer likes both and has a diminishing marginal rate of substitution of hot dogs for chili)**



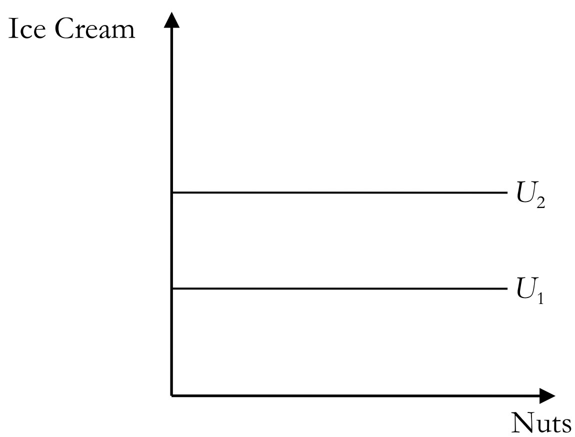
   b. **Sugar and Sweet'N Low (the consumer likes both and will accept an ounce of Sweet'N Low or an ounce of sugar with equal satisfaction)**



c. Peanut butter and jelly (the consumer likes exactly 2 ounces of peanut butter for every ounce of jelly)



d. Nuts (which the consumer neither likes nor dislikes) and ice cream (which the consumer likes)



4. **Consider the utility function $U(x, y) = 3x + y$.**

a) **Is the assumption that more is better satisfied for both goods?**

Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) **Calculate the marginal utility of X. Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x? Explain.**

The marginal utility of x remains constant at 3 for all values of x.

c) **Calculate the $MRS_{x,y}$ and interpret it in words**

MRSx,y = MUx/MUy = 3/1 = 3

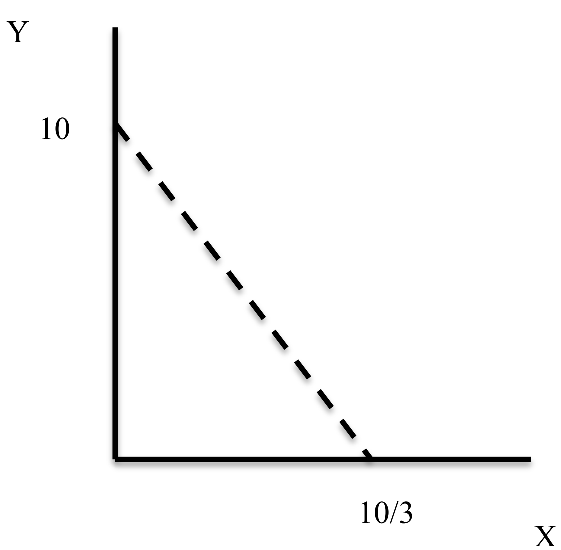
Remember the slope is dY/dX. This means that the consumer is willing to drive of 1 X for every 3 Y

d) **Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?**

MRS remains constant

e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.

Here is an indifference curve for U=10. Yes indifference curve intersects both axes



5. Ch 3, Problem 3.16

Consider the utility function $U(x, y) = X^{1/2}Y^{1/2}$

a) **Is the assumption that more is better satisfied for both goods?**

Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

b) **Calculate the marginal utility of X. Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more x? Explain.**

$$MUx = \frac{\partial U}{\partial X} = \frac{1}{2}\frac{Y^{1.2}}{x^{1/2}}$$

So as you consume more X (and less Y), the Mux diminishes

c) **Calculate the $MRS_{x,y}$ and interpret it in words**

$$MRSx,y = \frac{MUx}{MUy} = \frac{\frac{1}{2}\frac{Y^{1/2}}{X^{1/2}}}{\frac{1}{2}\frac{X^{1/2}}{Y^{1/2}}} = \frac{Y}{X}$$

Will to give up 1 Y for 1 X and still be on the same indifference curve.

d) **Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?**

As X increases (and Y decreases) as we move right along the indifference curve the MRS is diminishing.

e) On a graph with x on the horizontal axis and y on the vertical axis, draw a typical indifference curve (it need not be exactly to scale, but it needs to reflect accurately whether there is a diminishing $MRS_{x,y}$). Also indicate on your graph whether the indifference curve will intersect either or both axes. Label the curve U_1.