Ch 8 Problem Set Solutions

1. A firm's long-run total cost curve is $TC(Q) = 40Q - 10Q^2 + Q^3$. Over what range of output does the production function exhibit economies of scale? Over what range does it exhibit diseconomies of scale? At what quantity is minimum efficient scale?

Economies of scale occur when $MC < AC$; diseconomies of scale occur when $MC > AC$; and minimum efficient scale occurs where $MC = AC$ (if MC is increasing at that point). Clearly, the first step is to calculate MC and AC:

$$MC(Q) = 40 - 20Q + 3Q^2$$

$$AC(Q) = 40 - 10Q + Q^2$$

Economies of scale occur when $MC < AC$:

$$40 - 20Q + 3Q^2 < 40 - 10Q + Q^2$$

$$2Q^2 - 10Q < 0$$

$$Q(2Q - 10) < 0$$

Hence, we have economies of scale when $Q < 5$.

Similarly, diseconomies of scale occur when $MC > AC$. Hence, we have diseconomies of scale when $Q > 5$.

And finally, since AC is decreasing (economies of scale) for $Q < 5$ and increasing (diseconomies of scale) for $Q > 5$, it must be the case that minimum AC occurs at $Q = 5$. Hence, the minimum efficient scale is $Q = 5$.

2. Ch 8 problem 8.10: For each of the total cost functions, write the expression for the total fixed cost, average variable cost, and marginal cost:

a. TC(Q)=10Q

$TFC = 0$, $AVC = 10$, $MC = 10$.

b. TC(Q)=160+10Q

$TFC = 160$, $AVC = 10$, $MC = 10$.

c. TC(Q)=10Q²

$TFC = 0, AVC = 10Q$, MC=20Q

d. TC(Q)=160+10Q$^{1/2}$

$TFC = 160, AVC = 10Q^{-1/2}$, MC=5/Q$^{-1/2}$

3. **Ch 8 problem 8.11: A firm produces a product with labor and capital as inputs. The production function is described by Q=LK. Let w=1 and r=1. Find the equation for the firm's long-run total cost curve as a function of quantity Q.**

From Question 3 on the last homework we know the input demand functions:

$$L = \sqrt{\frac{rQ_0}{w}}$$

$$K=\sqrt{\frac{wQ_0}{r}}$$

We sub these back into the total cost function:

$$TC = w\sqrt{\frac{rQ_0}{w}} \quad + r\sqrt{\frac{wQ_0}{r}}$$

$$TC = \sqrt{wrQ_0} \quad + \sqrt{wrQ_0} = 2\sqrt{wrQ_0}$$

when r=1 and w=1

$$TC = 2\sqrt{Q_o}$$

4. Ch 8 problem 8.13: A firm produces a product with labor and capital. Its production function is described by Q=L+K. Let w=1 and r=1 be the prices of labor and capital respectively.

a. Find the equation for the firm's long-run total cost curve as a function of quantity Q when the prices of labor and capital are w=1 and r=1.

With a linear production function, the firm operates at a corner point, so will use all labor or all capital. We want to compare MP_L/w with MP_k/r. This tells us for another dollar spent on labor how much more product/output would be produced, or for a dollar spent on capital how much more product/output would be produced. Because the marginal products and the prices of the inputs are exactly the, the firm is indifferent among combination of *L* and *K*.

b. Find the solution to the firm's short-run cost-minimization problem when capital is fixed at a quantity of 5 units (i.e. K=5), and w=1 and r=1. Derive the equation for

the firm's short-run total cost curve, average cost curve and marginal cost curve as a function of Q.

When capital is fixed at 5 units, the firm's output would be given by $Q = 5 + L$. If the firm wants to produce $Q < 5$ units of output, it must produce 5 units and throw away $5 - Q$ of them. The total cost of producing fewer than 5 units is constant and equal to \$5, the cost of the fixed capital. For $Q > 5$ units, the firm increases its output by increasing its use of labor. In particular, using the production function we can see that to produce Q units of output, the firm uses $Q - 5$ units of labor (L=Q-5 when we sub in for K=5). Subbing L and K into the Total cost function we get:, $STC(Q) = Q - 5 + 5 = Q$